Possible Cosmological Implications of the Quark-Hadron Phase Transition

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Abstract.

We study the quark-hadron phase transition within an effective model of QCD, and find that in a reasonable range of the main parameters of the model, bodies with quark content between 10^{-2} and 10 solar masses can have been formed in the early universe. In addition, we show that a significant amount of entropy is released during the transition. This may imply the existence of a higher baryon number density than what is usually expected at temperatures above the QCD scale. The cosmological QCD transition may then provide a natural way for decreasing the high baryon asymmetry created by an Affleck-Dine like mechanism down to the value required by primordial nucleosynthesis.

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1. Introduction

It has been shown that the quark-hadron phase transition which occurred in the early Universe could lead to the formation of relic quark-gluon plasma objects, which survive today [1, 2]. Generally, it is admitted [1, 3, 4, 5] that the transition occurred effectively at the critical temperature, which is of the order 100 MeV, the QCD energy scale. In that case, the quark content of the bodies which have been formed during the transition cannot be larger than 10^{-8} M$_\odot$. Another possibility arises if the transition was delayed for some time. It then becomes possible that the quark plasma objects formed at the end of the transition appeared at a temperature much lower than T_c, and more massive bodies may have been produced [6, 7, 8]; these latter could then account for some fraction of the dark matter in the Universe.

In this work, we perform a detailed analysis of the phase transition within an effective model of QCD, and we show that for the same critical temperature as what is usually thought, we can obtain a high degree of supercooling, which allows the possible formation of large "quark stars" with masses ranging from 10^{-2} to 10 M$_\odot$. We find that this is so because the Universe has grown exponentially during the quark-hadron transition, at a temperature $T \lesssim 100$ MeV, much lower than the temperature of classical inflationary models [9]. Moreover, we show that the exponential expansion is not balanced by so steep a drop of temperature, and thus increases significantly the total entropy of the Universe. This entropy production dilutes the density of any conserved or quasi-conserved quantity present before the transition, such as the baryon number. Therefore, our model requires a high value of these quantities at $T \gtrsim T_c$, as for example the large baryon asymmetry produced in some supersymmetric models [10]. This leads us to incorporate the quark chemical potential in our model, and to study the cosmological quark-hadron transition in the context of both high temperature and quark number density.

2. The Model

Except in the quenched approximation, the theory of strong interactions QCD is too difficult, computationally, to give a reliable description of the phase transition between the high temperature and density phase of the quark gluon plasma and the low temperature and density phase of quarks bound in hadrons. In fact, lattice calculations [11] do not seem able yet to really tackle the problem of finite quark densities. Because the light quarks have such a small mass compared with the transition temperature or chemical potential, we believe them to play an essential role in determining the quantitative features of the transition which is not taken into account in the quenched approximation, as shows for example the discrepancy between the values of the critical temperature computed with or without dynamical quarks. In considering the transition, we therefore work with a simple model in which non-perturbative QCD is replaced by an effective Lagrangian which incorporates the chiral symmetry of the theory. In this

model the quark fields are interacting with a chiral field formed with the π meson field and the scalar field σ. The Lagrangian density is

$$\mathcal{L} = \sum_{k=1}^{n_{\rm f}} \left[i\bar{\psi}_k \gamma^\mu \partial_\mu \psi_k - g\bar{\psi}_k(\sigma + i\tau \cdot \pi\gamma_5)\psi_k \right] + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi - V(\sigma^2+\pi^2) \quad (1)$$

which can be rewritten as [12]

$$\mathcal{L} = \sum_{k=1}^{n_{\rm f}} \left[i\bar{\psi}_k \gamma^\mu \partial_\mu \psi_k - g\xi(\bar{\psi}_k^{\rm L} U \psi_k^{\rm R} + \bar{\psi}_k^{\rm R} U^+ \psi_k^{\rm L}) \right] + \frac{1}{2}\partial_\mu\xi\partial^\mu\xi + \frac{1}{4}\xi^2 \ {\rm Tr}(\partial_\mu U \partial^\mu U^+) - V(\xi) \quad (2)$$

where $\psi_k^{\rm L,R}$ are the left- and right-handed components of the quark field ψ_k, U is an element of SU(2) defined by $\xi U = \sigma + i\tau \cdot \pi$ and $\xi = (\sigma^2+\pi^2)^{\frac{1}{2}}$.

The generalized self-interaction potential is the usual quartic function in ξ. We choose to write it in a seemingly complicated form, but such that the parameters f_π, λ, B are readily related to physical quantities.

$$V(\xi) = \frac{1}{2} f_\pi^2 \left(\lambda^2 - \frac{12B}{f_\pi^4} \right) \xi^2 \left(1 - \frac{\xi}{f_\pi} \right)^2 + B \left[1 + 3\left(\frac{\xi}{f_\pi}\right)^4 - 4\left(\frac{\xi}{f_\pi}\right)^3 \right] \quad (3)$$

$V(\xi)$ has its absolute minimum at $\xi = f_\pi = 93$ MeV, a value chosen to fit the observed pion decay rate. For this value, which corresponds to the physical vacuum, chiral symmetry is spontaneously broken; however, if $\pi = 0$ isospin symmetry is preserved, just as in nature. If $B < \frac{\lambda^2 f_\pi^4}{12}$, the potential has a second local minimum at $\xi = 0$, corresponding to a chirally symmetric, metastable "false vacuum" with the energy density B. This is analogous to the perturbative vacuum of the MIT bag model [13], and B can be interpreted as the bag constant in that model. Various versions of the previous model have often been used to study the low energy hadron spectroscopy [14] or heavy ion collisions [15], and phenomenological fits to light hadron properties give $B^{1/4}$ between 100 and 200 MeV [16]. Small oscillations of ξ about the minimum at $\xi = f_\pi$ correspond to a scalar particle of mass $m_\xi = \lambda f_\pi$ and three massless pseudoscalar pions. We can notice that the ξ field is a chiral singlet, and the associated scalar particle may be interpreted as representing the condensate arising from non-linear interactions of the gluons (glueballs). As the lightest glueballs are believed to have a mass in the range 1.5 to 1.7 GeV [17, 18], λ is between 16 and 19. The coupling constant g gives mass to the quarks, and thus helps to break the chiral symmetry; following the fits to hadronic physics, we take it to be larger than 10, so that quarks have an effective mass larger than 1 GeV in the physical vacuum: it is then energetically unfavourable for them to exist in the phase with $\xi = f_\pi$. However, the actual value of g is not important for the following calculations.

In order to study the transition which took place in the early Universe, at a temperature of the order 100 MeV, we must implement finite temperature in our model. According to the common lore, it seems irrelevant to take into account a possible chemical potential, because the quark density in the cosmological context is too small to be significant (see e.g. [19], pp. 530-531). To this effect, we must add to the potential

given by equation (3) contributions like [20, 21]

$$f_f(T) = \, -2T \int \frac{d^3k}{(2\pi)^3} \ln\left(1 + e^{-E(k)/T}\right), \tag{4}$$
$$E(k) \, = \sqrt{k^2 + g^2\xi^2}$$

for each fermionic degree of freedom (spin, flavour, colour), and

$$f_b(T) = T \int \frac{d^3k}{(2\pi)^3} \ln\left(1 - e^{-E'(k)/T}\right), \tag{5}$$
$$E'(k) = \sqrt{k^2 + m_\xi^2}$$

for bosonic degrees of freedom. These terms come from the one loop approximation and represent the effect of the thermal excitations of the quark-antiquark pairs and of the ξ field. These latter have a mass $\left(\frac{\partial^2 V}{\partial \xi^2}\right)^{1/2}$ larger than m_ξ in both phases with $\xi = 0$ and $\xi = f_\pi$, and are strongly suppressed by the Boltzmann factor $e^{-E'/T}$ at $T \lesssim 100$ MeV. The same arguments afford us to discard the excitations of q$\bar{\text{q}}$ pairs in the physical vacuum $\xi = f_\pi$, but not in the false vacuum $\xi = 0$, where the contributions given by equation (4) lower the value of the effective potential. For ξ close to 0, which implies $\frac{g\xi}{T} \ll 1$, the integral can be expanded as [20, 22]

$$f_f(T) = -\frac{7\pi^2}{360} T^4 + \frac{1}{24} g^2 \xi^2 T^2 \tag{6}$$

Actually, we have checked numerically that equation (6) represents a very accurate approximation. We include in our model the three light quarks. Each contribute for 3 degrees of freedom of colour, and 2 of spin, giving 18 degrees of freedom in all. The u and d quarks are considered massless, whereas for the strange quark we will keep the quadratic term of equation (6) to take into account its finite mass.

With these modifications, the self-interaction potential at finite temperature and $\xi = 0$ can be rewritten

$$V_T(\xi) = B - \alpha_T T^4 + \gamma_T T^2 \tag{7}$$

where $\alpha_T = \frac{7\pi^2}{20}$ and $\gamma_T = \frac{1}{4} m_s^2$, $m_s = (60 - 170)$ MeV.[18]

A sketch of $V_T(\xi)$ for different values of the temperature is given in figure 1. It has two minima, at $\xi \simeq f_\pi$ with the same value as at zero temperature, and at $\xi = 0$, where the value of the potential is lowered. The overpressure, difference in free energy density between these minima, is

$$\Delta P = B + \gamma_T T^2 - \alpha_T T^4 \tag{8}$$

This difference vanishes when

$$T = T_c = \sqrt{\frac{\gamma_T + \sqrt{\gamma_T^2 + 4\alpha_T B}}{2\alpha_T}} \tag{9}$$

For this value, both minima of $V_T(\xi)$ are degenerate, which gives a first order phase transition. Even though the order of the QCD transition is still a highly debated issue,

this is in agreement with previous theoretical predictions using the linear σ model [23], which, incidentally, is a particular case of our model when $\lambda^2 f_\pi^4 = 8B$, or with recent indications from lattice calculations with 3 degenerate Wilson quarks or with 2 massless quarks and a light s quark [24]. T_c is then the critical temperature of the transition. If $T > T_c$, the minimum at $\xi = 0$ becomes the absolute minimum of $V_T(\xi)$.

Besides the quarks, we first include in our model a thermal bath of relativistic particles which influence the transition only through gravitational effects: at this stage, they represent the "physical vacuum". The contribution to the free energy density of both phases of these spectator particles (photons, electrons, muons and neutrinos) is given by expressions (4) and (5) for fermions with their antiparticles and bosons respectively. These terms can be expanded as $f_f = -\frac{7\pi^2}{360}T^4 + \frac{1}{24}m^2T^2$, with a mass term m_μ only for the muon, and $f_b = -\frac{\pi^2}{90}T^4$ for the massless photon, so we get an extra contribution

$$f_v = -\frac{14.25\pi^2}{90}T^4 + \frac{1}{12}m_\mu^2 T^2 = -\alpha_v T^4 + \gamma_v T^2 \tag{10}$$

contributing to both phases. However, if we consider the overpressure ΔP between the phases with $\xi = 0$ and $\xi = f_\pi$, this extra free energy density cancels, and ΔP is still given by equation (8).

In a more exhaustive calculation, we will also include in the hadronic phase $\xi = f_\pi$ the lightest hadrons, the pions, and show that their influence on the results, which we discuss later, is not qualitatively important.

3. Bubble nucleation. Dynamics of the Universe

After the Big Bang, the temperature of the early Universe is higher than the critical temperature of the quark-hadron transition (equation 9): the phase with $\xi = 0$ is the more stable. The massless quarks are deconfined – this is the so-called quark-gluon plasma –, and chiral symmetry is preserved. According to the standard model of cosmology, the Universe expands, while its temperature decreases. When it drops below T_c, bubbles of the true vacuum $\xi = f_\pi$, which is now the stable phase, begin to appear within the quark plasma. If we consider only homogeneous nucleation, the nucleation rate per unit volume is given by [25]

$$\Gamma(T) = CT^4 \left(\frac{S_3}{2\pi T}\right)^{3/2} e^{-S_3/T} \tag{11}$$

C is a multiplicative coefficient of the order unity [26]. For a spherical bubble, S_3 is the stationary value of the functional $F = 4\pi \int r^2 \left[\frac{1}{2}(\frac{d\xi}{dr})^2 + V_T(\xi)\right] dr$. In the thin-wall approximation [25], this expression can be replaced by $F = -\frac{4}{3}\pi r_0^3 \Delta P + 4\pi r_0^2 s$, where r_0 is the bubble radius, the overpressure ΔP is the difference in $V_T(\xi)$ between the two minima, given by expression (8), and s is the surface tension. This latter is

$$s = \int_0^{f_\pi} \sqrt{2V_{T_c}(\xi)}d\xi = \lambda f_\pi^3 I(y) \tag{12}$$

with [7]

$$I(y) = \int_0^1 (1-u)\sqrt{u^2 + \frac{y}{6}(1+2u-3u^2)}\, du \ , \quad y = \frac{12B}{\lambda^2 f_\pi^4}. \tag{13}$$

The graph of $I(y)$ in figure 2 shows that $\frac{1}{6} < I(y) < \frac{1}{3}$. It should be noted that s is entirely fixed by the parameters of the zero-temperature model, determined at low energy. Its value is then $50 \leq s \leq 120$ MeV/fm^2, higher than the estimates given by lattice computations without dynamical quarks [27], but in agreement with the surface tension obtained by Burakovsky [28] within an effective model of QCD different from ours.

F is stationary at a radius $r_0 = \frac{2s}{\Delta P}$, and takes the value

$$S_3 = \frac{16\pi}{3}\frac{s^3}{\Delta P^2} = \frac{16\pi}{3}\frac{\left[\lambda f_\pi^3 I(y)\right]^3}{\Delta P^2} \tag{14}$$

Let us call t_c the time at which the temperature equals T_c. When $t \gtrsim t_c$, both phases with $\xi = 0$ and $\xi = f_\pi$ coexist. If $x(t)$ is the fraction of the Universe occupied by the physical vacuum, and therefore $1 - x(t)$ the fraction taken up by the quark plasma, the energy density of the matter which fills the Universe is

$$\begin{aligned} \varepsilon(t) &= [1-x(t)]\varepsilon_{\xi=0}(t) + x(t)\varepsilon_{\xi=f_\pi}(t) \\ &= [1-x(t)]\left[\varepsilon_q(t) + \varepsilon_v(t)\right] + x(t)\varepsilon_v(t) \end{aligned} \tag{15}$$

ε_q, ε_v, $\varepsilon_{\xi=0}$ and $\varepsilon_{\xi=f_\pi}$ stand respectively for the energy densities of the quark-gluon plasma, the thermal bath of photons and leptons, the metastable vacuum and the physical vacuum. They are given by

$$\varepsilon_q = B - \gamma_T T^2 + 3\alpha_T T^4 \tag{16}$$

$$\varepsilon_v = 3\alpha_v T^4 - \gamma_v T^2 \tag{17}$$

$$\varepsilon_{\xi=0} = \varepsilon_q + \varepsilon_v$$

$$\varepsilon_{\xi=f_\pi} = \varepsilon_v$$

Expression (15) can be rewritten

$$\varepsilon(t) = [1-x(t)]\varepsilon_q(t) + \varepsilon_v(t) \tag{18}$$

It appears as the sum of two contributions corresponding to the quark plasma, which is going to disappear during the phase transition, and to the background of non-interacting particles. If we replace the energy densities $\varepsilon_q(t)$ and $\varepsilon_v(t)$ with their temperature-dependent expressions (16, 17), we obtain

$$\varepsilon(t) = [1-x(t)]\left[B - \gamma_T T(t)^2 + 3\alpha_T T(t)^4\right] + 3\alpha_v T(t)^4 - \gamma_v T(t)^2 \tag{19}$$

If we now suppose that the nucleation of the bubbles of phase with $\xi = f_\pi$ is isotropic enough, the Universe is still described by the Robertson-Walker metric. We can then deduce from Friedmann's equation and the previous expression the equation which governs the expansion :

$$\frac{1}{R(t)}\frac{dR}{dt}(t) = \sqrt{\frac{8\pi}{3m_P^2}\varepsilon(t)} \tag{20a}$$

$$\frac{dR}{Rdt} = \sqrt{\frac{8\pi}{3m_P^2}\left[(1-x)\left(B-\gamma_T T^2+3\alpha_T T^4\right)+3\alpha_v T^4-\gamma_v T^2\right]} \qquad (20b)$$

where m_P is the Planck mass while x and T are time-dependent.

In the beginning, few bubbles appear, and the temperature still decreases. As time goes on, the size of bubbles increases, because of both the propagation of the bubble walls and the expansion of the Universe. For example, if we consider a bubble of radius $r(t)$ which appeared at time t_i, its growth is given by $\frac{dr}{dt} = v + r\frac{1}{R}\frac{dR}{dt}$, hence

$$r(t) = vR(t)\int_{t_i}^{t}\frac{dt'}{R(t')} \qquad (21)$$

v is the propagation speed of the walls, which we shall take equal to the velocity of light in our calculations.

While the size of existing bubbles increases, new bubbles form. Thus the number density of nucleation sites at time t is

$$N(t) = \int_{t_c}^{t}\left[1-x(t')\right]\Gamma(t')\left[\frac{R(t')}{R(t)}\right]^3 dt' \qquad (22)$$

This expression shows that the number $N(t)[R(t)]^3$ of bubbles in a unit comoving volume which exist at t, i.e. those which appeared at any time $t' < t$, is increasing. However, the number of bubbles per unit physical volume $N(t)$ may either increase, when the nucleation rate $\Gamma(t')$ is large, or decrease, when the expansion factor $\left[\frac{R(t)}{R(t')}\right]^3$ becomes large. In other words, the mean physical distance between nucleation sites may either decrease, in the first case, or increase as the Universe expands. Nonetheless, the fraction $x(t)$ of the Universe filled up with physical vacuum is growing with the number of bubbles. As we will see later, it is necessary to take into account the expansion of the Universe in the evolution of this fraction, so that

$$x(t) = 1-\exp\left(-\int_{t_c}^{t}\left[1-x(t')\right]\Gamma(t')\frac{4\pi}{3}\left[vR(t)\int_{t'}^{t}\frac{dt''}{R(t'')}\right]^3 dt'\right) \qquad (23)$$

This expression is analogous to the formula derived by Guth and Weinberg [29] in the context of GUT phase transitions, with the exception of the extra factor $1-x(t')$, which takes into account the fact that hadronic bubbles can only appear within the quark plasma.

The evolution of T during the phase transition is easily determined by using the energy conservation, which reads

$$\left[\varepsilon(t)+P(t)\right]\frac{dR^3}{dt} + R(t)^3\frac{d\varepsilon}{dt} = 0 \qquad (24)$$

$\varepsilon(t)$ is the energy density, given by expression (19), whereas the pressure is

$$P = (1-x)(\alpha_T T^4-\gamma_T T^2-B)+\alpha_v T^4-\gamma_v T^2 \qquad (25)$$

The time-dependences of ε or P are connected with those of T and x, so we get

$$\frac{dT}{dt} = -\frac{(1-x)\,(6\alpha_T T^3 - 3\gamma_T T) + 6\alpha_v T^3 - 3\gamma_v T}{(1-x)\,(6\alpha_T T^2 - \gamma_T) + 6\alpha_v T^2 - \gamma_v}\frac{dR}{Rdt} + \frac{3\alpha_T T^4 - \gamma_T T^2 + B}{(1-x)\,(12\alpha_T T^3 - 2\gamma_T T) + 12\alpha_v T^3 - 2\gamma_v T}\frac{dx}{dt} \tag{26}$$

$\frac{dT}{dt}$ is the sum of two terms : the first one is the usual contribution of the expansion of the Universe, which decreases the temperature. The second contribution comes from the replacement of the false vacuum $\xi = 0$ with the physical vacuum $\xi = f_\pi$, which has a much lower specific heat, and corresponds to the release of latent heat, which tends to increase T. In earlier studies [1, 3, 4, 5], it is assumed that the latent heat released by the first bubbles of hadronic phase formed when the temperature drops under T_c quickly reheats the Universe up to the critical temperature, and further bubble nucleation is suppressed. For the remainder of the transition, i.e. some microseconds, the temperature is kept constant by the latent heat due to the growth of the hadronic bubbles, which balances the expansion of the Universe. This latter remains negligible: equation (21) reduces to $r(t) \approx v(t - t_c)$, while the expansion factor in expression (22) is almost equal to 1. In this work, we investigate this point more carefully, following the detailed evolution of the temperature. In particular, we will allow out-of-equilibrium coexistence at a temperature different from T_c between the quark plasma and the hadronic phase. For example we can expect from equation (26) that the product $R(t)T(t)$ will not remain constant during the transition, and the total entropy of the Universe, which is proportional to $(RT)^3$, will not be conserved.

In many calculations concerning first order cosmological phase transitions [29, 30], it is assumed that $R(t)T(t) =$ constant during the transition, i.e. that entropy is conserved. In that case, the second term of equation (26) is neglected, and following the work of Coleman [31] it is thought that all the latent heat is taken by the bubble walls. At the completion of the transition the Universe would then reheat as the energy in the walls is dissipated by their collisions [32], and T would quickly increase up to a temperature close to its initial value T_c. In this work we will see that there is no such steep drop of temperature, but that the latent heat is released gradually, and holds T rather high, though not necessarily at T_c, during the entire transition. When this latter is completed, the temperature may then be significantly smaller than the critical temperature T_c.

After a while, bubbles begin to coalesce and to enclose lumps of the phase with $\xi = 0$. The quark number enclosed in such a chunk is conserved, whereas the size of the lump decreases as the bubble walls propagate, and therefore its density increases, until the pressure of the quark plasma trapped inside balances the pressure of the physical vacuum $\xi = f_\pi$. At that time, we can consider that $x \simeq 1$, because the regions filled with quark plasma are but a minute fraction of the Universe, and the transition has come to an end. Let us call t_f the time at which $x(t_f) = 1$, and $N(t_f)$ the corresponding number density of nucleation sites. To simplify the calculation we have not considered the dispersion in the number of trapped quarks in the chunks as given by the statistical distribution of nucleation sites. To estimate the mean number of quarks per body, we

shall assume that these sites are on the vertices of a cubic lattice. At the centre of each cube is a quark clump. If no phase transition had occurred, the quarks enclosed would take up a volume $V_f = \frac{1}{N(t_f)}$, which corresponds to a physical volume $V_c = V_f \left[\frac{R(t_c)}{R(t_f)}\right]^3$ at the beginning of the transition. Therefore, the body contains $\mathcal{N}_q = n_q(t_c)V_c$ quarks, where $n_q(t)$ is the quark number density at time t, which can be easily related to the photon number density $n_\gamma = \frac{2}{\pi^2}\zeta(3)T^3$ and the quark to photon ratio $\frac{n_q}{n_\gamma}$. The conservation of the quark number when there is no baryon number violating interaction reads $n_q(t_c)[R(t_c)]^3 = n_q(t_f)[R(t_f)]^3$, so it is possible to rewrite $\mathcal{N}_q = n_q(t_f)V_f = \frac{n_q(t_f)}{N(t_f)}$, hence

$$\mathcal{N}_q = \frac{2}{\pi^2}\zeta(3)\frac{n_q}{n_\gamma}(t_f)\frac{T_f^3}{N(t_f)} \tag{27}$$

$\frac{n_q}{n_\gamma}(t_f)$ is the quark to photon ratio at the end of the transition. We suppose that this value remained unchanged till the beginning of the Big Bang Nucleosynthesis at $T \simeq 1$ MeV, and therefore take in our calculations $\frac{n_q}{n_\gamma} = \frac{3n_B}{n_\gamma} \simeq 10^{-9}$, with n_B the baryon number density [33]. As a matter of fact, this value of $\frac{n_q}{n_\gamma}$ is valid in the hadronic phase, not in the quark plasma. However, estimates [1, 4] show that the baryon number density in the quark phase should be larger than in the hadron phase, and thus our assumption tends to underestimate $\mathcal{N}_q$.

4. Results

We have tested our model with different values of the parameters B, m_s and λ. With a high value of B, such as $B^{1/4} = 200$ MeV, and $m_s = 100$ MeV, the critical temperature is $T_c \simeq 148$ MeV. If we take $\lambda = 18$, we find the usual results [1, 5], although our model is different, and the number of quarks trapped in a nugget at the end of the transition, which is completed in some microseconds, is $\mathcal{N}_q \simeq 10^{40}$. In that case, the standard model of cosmology shows that the horizon at t_f is too small to allow any object with a mass exceeding 10^{-8} $M_\odot$ to be formed [22, 34], which is in agreement with what we find. However, it may be worth noting that the upper limit given by this simple but powerful argument will no longer be valid if the Hubble volume is larger than what is given by the strictest standard model of cosmology (i.e. $R(t) \propto t^{1/2}$).

Another limiting situation appears when B takes a very low value, as for example $B^{1/4} = 50$ MeV. In this case, we find that $x(t)$ cannot reach 1, and therefore the transition never ends. The energy density which confines quarks within solitons is so weak that few bubbles of the physical vacuum $\xi = f_\pi$ are nucleated, and they are so far away from each other that their walls cannot meet while the Universe expands. An analogous phenomenon occurs when the surface tension s is too high. This happen also if the mean distance between bubbles $[N(t_f)]^{1/3}$ is smaller i.e. if the value of B is larger, but if simultaneously the speed of the bubble walls v is decreased. Once again, the expansion of the bubbles cannot catch up with the expansion of the Universe.

Now if B takes intermediate values, like $B^{1/4} = 120$ MeV (i.e., in other units, $B = 0.61$ fm^{-4}, which is obtained from fits to hadronic properties in [35]) with

$m_s = 100$ MeV, which gives a critical temperature $T_c = 90.1$ MeV, and $\lambda \simeq 17.77$, corresponding to a glueball mass of 1.6 GeV, the number of quarks enclosed is $\mathcal{N}_q \simeq 10^{57}$, i.e. a quark content of approximately 0.3 $M_\odot$. We have plotted the evolution during the transition of several quantities in such a case: the fraction $x(t)$ of Universe filled with physical vacuum, the temperature $T(t)$ and the scale factor $R(t)$.

We can see in figure 3 that $x(t)$ remains close to 0 for some time, when few bubbles have yet appeared. Then it rises quickly, though not instantaneously as in the "fast" scenario, and tends towards its asymptotical value 1. The time scale of the transition is now of the order of the millisecond, rather than the microsecond.

The temperature (figure 4) behaves as we had foretold. In the beginning, it decreases with a law close to the $t^{-1/2}$ law of the standard model: the first term of the r.h.s. of equation (26) is dominant. Then it increases, though not much, when one phase is replaced with another: the influence of the term in $\frac{dx}{dt}$, which represents the release of latent heat, is the more important; we can check that the fast growth of x and the increase in T are simultaneous. Finally, T decreases like $t^{-1/2}$ again, when the Universe is mostly filled up with physical vacuum, down to $T_f = 17.7$ MeV.

The scale factor R is plotted in figure 5. Obviously, it does not follow the $t^{1/2}$ law of the standard model, but rather seems to grow exponentially, to end up with a value $R_f \simeq 7 \times 10^3 R_c$. It should be noted that in this case, the dilution factor in equation (22) is no longer negligible, since $\left[\frac{R(t')}{R(t)}\right]^3$ can be as small as 10^{-11}! During the transition, the scale factor has been multiplied by $\simeq 10^4$, and therefore the Hubble radius is 10^4 times the radius predicted in the strict standard model: the Hubble volume is $\simeq 10^{11}$ times larger than what is expected, and there is no contradiction between the number $\mathcal{N}_q$ of quarks enclosed in a nugget and the total number of quarks within the Hubble volume.

If we compare our model with Guth's inflationary model [30], we can trace whence such a growth comes. As in his case, when the temperature has dropped sufficiently, its contribution in our equations (eq. 19) can be neglected with regard to the metastable vacuum energy density B. This latter then drives the expansion [36]. However, in our model the exponential growth comes to a natural end according to equation (20b), since the factor $1 - x(t)$ vanishes when the transition is completed. A second difference between our model and old inflation *à la* Guth regards entropy. In his model entropy is conserved during the exponential expansion and increases, due to reheating when bubbles collide, only at the completion of the transition. On the contrary, here entropy is constantly increasing during the quark-hadron phase transition, as the product $R(t)T(t)$ does not remain constant. To be more precise, the scale factor R is multiplied by 7×10^3, whereas T only decreases from 90 to 18 MeV: the product RT is multiplied by more than 1.4×10^3, and hence the entropy increases by a factor 3×10^9.

This increase in entropy has radical consequences on the evolution of the ratio $\frac{n_q}{n_\gamma}$, which is a measure of the baryon asymmetry of the Universe, during the transition. We have already seen that if there is no baryon number violating interaction, the quark number density n_q decreases as R^{-3}, and that $n_\gamma \propto T^3$; hence, the ratio $\frac{n_q}{n_\gamma} \propto (RT)^{-3}$ is

proportional to the inverse of the entropy. In particular, if, as we have found before, the entropy is multiplied by 10^9 during the quark-hadron transition, the baryon asymmetry is divided by the same factor. Therefore, if at the end of the transition $\frac{n_q}{n_\gamma} \simeq 10^{-9}$ as required by primordial nucleosynthesis, it means that at $T \gtrsim T_c$ the ratio was of the order 1. The results found here contrast sharply with normal adiabatic expansion in which this baryon asymmetry would not change. However, when $\frac{n_q}{n_\gamma} \simeq 1$, the approximation made so far of neglecting the quark density or the chemical potential of the quarks with regard to T is no longer valid.

5. Finite chemical potential

To handle the possibility of large quark densities, we must now incorporate chemical potential into our model. Within the one loop approximation, the contributions of fermionic degrees of freedom which should be added to the effective potential $V(\xi)$ at zero temperature and zero chemical potential are no longer given by expression (4), but by

$$\omega_f(T,\mu) = -T\left[\int \frac{d^3k}{(2\pi)^3}\ln\left(1+e^{-(E(k)-\mu)/T}\right) + \int \frac{d^3k}{(2\pi)^3}\ln\left(1+e^{-(E(k)+\mu)/T}\right)\right] \tag{28}$$

The first term is the fermion contribution, the second is from the antifermions. We have changed the notation from f_f to ω_f because at finite μ, the free energy and the thermodynamic potential do not coincide any more. For the same reason as before, the excitations of the scalar field vanish at both minima of $V(\xi)$, whereas those of quark-antiquark pairs are important only for ξ close to 0, where it becomes possible to expand the right-hand side of equation (28) as

$$\omega_f(T,\mu) = -\frac{7\pi^2}{360}T^4 - \frac{1}{12}\mu^2T^2 - \frac{1}{24\pi^2}\mu^4 + \frac{1}{24}g^2\xi^2T^2 + \frac{1}{8\pi^2}g^2\xi^2\mu^2 \tag{29}$$

As previously, we keep the mass terms only for the strange quark. The self-interaction potential at $\xi = 0$ reads

$$V_{T,\mu}(\xi) = B - \alpha_T T^4 - \gamma_{\mu T}\mu^2T^2 - \alpha_\mu\mu^4 + \gamma_T T^2 + \gamma_\mu\mu^2 \tag{30}$$

$$\alpha_T = \frac{7\pi^2}{20}, \quad \gamma_{\mu T} = \frac{3}{2}, \quad \alpha_\mu = \frac{3}{4\pi^2}, \quad \gamma_T = \frac{1}{4}m_s^2, \quad \gamma_\mu = \frac{3}{4\pi^2}m_s^2$$

As before, we consider B and m_s as parameters, while μ is related to the quark number density by $n_q = 4\alpha_\mu\mu^3 + 2\gamma_{\mu T}\mu T^2 - 2\gamma_\mu\mu$.

Therefore, the difference in pressure between the two minima of $V_{T,\mu}(\xi)$ is

$$\Delta P = B + \gamma_T T^2 + \gamma_\mu\mu^2 - \alpha_T T^4 - \gamma_{\mu T}\mu^2T^2 - \alpha_\mu\mu^4 \tag{31}$$

This in turn changes the expression of the critical temperature, given by the condition $\Delta P = 0$, and of the bubble nucleation rate (11).

The energy density which drives the expansion of the Universe through Friedmann's equation $(20a)$ is now

$$\varepsilon = (1-x)\left(B - \gamma_T T^2 - \gamma_\mu\mu^2 + 3\alpha_T T^4 + 3\gamma_{\mu T}\mu^2T^2 + 3\alpha_\mu\mu^4\right) + 3\alpha_v T^4 - \gamma_v T^2 \tag{32}$$

whereas expression (26), which gives the evolution of temperature during the transition, becomes

$$\frac{dT}{dt} = -\frac{3T\left[(1-x)\left(2\alpha_T T^2 + \gamma_{\mu T}\mu^2 - \gamma_T - \frac{\gamma_{\mu T}\mu n_q}{6\alpha_\mu\mu^2+\gamma_{\mu T}T^2-\gamma_\mu}\right) + 2\alpha_v T^2 - \gamma_v\right]}{(1-x)\left(6\alpha_T T^2 + \gamma_{\mu T}\mu^2 - \gamma_T - \frac{4\gamma_{\mu T}^2\mu^2T^2}{6\alpha_\mu\mu^2+\gamma_{\mu T}T^2-\gamma_\mu}\right) + 6\alpha_v T^2 - \gamma_v}\frac{dR}{Rdt}$$
$$+\frac{B + 3\alpha_T T^4 + 3\alpha_\mu\mu^4 + 3\gamma_{\mu T}\mu^2T^2 - \gamma_T T^2 - \gamma_\mu\mu^2}{2T\left[(1-x)\left(6\alpha_T T^2 + \gamma_{\mu T}\mu^2 - \gamma_T - \frac{4\gamma_{\mu T}^2\mu^2T^2}{6\alpha_\mu\mu^2+\gamma_{\mu T}T^2-\gamma_\mu}\right) + 6\alpha_v T^2 - \gamma_v T^2\right]}\frac{dx}{dt} \qquad (33)$$

The remainder of the calculation of the number $\mathcal{N}_q$ of quarks trapped in a typical chunk is changed accordingly. However, whereas the number of the model parameters B, m_s and λ remains the same as in the vanishing chemical potential case, there is now an important extra requirement, namely that the quark to photon ratio after the transition should be equal to 10^{-9}, in agreement with the value required for primordial nucleosynthesis. From now on, we choose to take $B^{1/4} = 120$ MeV and $m_s = 100$ MeV, whereas λ will be determined by the above condition. In other words, once 2 of our 3 parameters have been fixed, for example the energy density of the false vacuum and the mass of the strange quark, the value of the third parameter cannot be chosen at will, but is dictated by the amount of entropy which has to be released during the transition. Hopefully, as we will see, the "good" values of λ come exactly in the range which gives the glueball mass.

If we take a chemical potential $\mu(t_c) = 31.44$ MeV, equivalent to $\frac{n_q}{n_\gamma}(t_c) = 1$ at the critical temperature $T_c = 89.96$ MeV, and with $\lambda \simeq 17.77$, at the end of the phase transition, the number of quarks enclosed in a chunk is $\mathcal{N}_q \simeq 2\times10^{57}$, i.e. approximately 0.5 $M_\odot$. The final temperature is $T_f = 17.91$ MeV and the ratio of the final and initial scale factors $\frac{R_f}{R_c} \simeq 4\times10^3$, so that the quark to photon ratio after the transition has become $\frac{n_q}{n_\gamma} \simeq 2\times10^{-9}$. However, we have to wonder how an important quark number density could have arisen in the early Universe, at temperatures above the QCD scale.

Affleck and Dine [10] have proposed a mechanism for baryogenesis which could give such a high initial value of $\frac{n_q}{n_\gamma}$. In their model, the decay when supersymmetry breaking effects become important of the large vacuum expectation value of a scalar fermion, acquired either by quantum fluctuations at the Planck epoch or after an episode of inflation [37], leads at a temperature of the order 10 TeV to a baryon asymmetry which can be as large as $\frac{n_q}{n_\gamma} = 10^3$. The actual value of the asymmetry depends on several parameters of the model, such as the expectation value of the squark field which decays or the CP-violating phase required to produce more quarks than antiquarks. After its production, this huge baryon asymmetry could still have been wiped out, long after the SUSY breaking epoch, by the so-called sphaleron transitions [38] arising, for example, from the electroweak anomaly. However, it was shown in reference [39] that a baryon-lepton number $(B-L)$† excess can be produced by the Affleck-Dine mechanism along with the usual B excess. This is sufficient to prevent any electroweak sphaleron induced

† Throughout this paragraph, B denotes the baryon number, not the energy density of the chirally symmetric vacuum $\xi = 0$

Table 1. Values of $\mathcal{N}_q$ with $B^{1/4} = 120$ MeV, $m_s = 100$ MeV, λ variable.

$\frac{n_q}{n_\gamma}(t_c)$	λ	T_c (MeV)	$\mu(t_c)$ (MeV)	T_f (MeV)	t_f (ms)	$\frac{R_f}{R_c}$	$\frac{n_q}{n_\gamma}(t_f)$	$\mathcal{N}_q$
0.01	17.77	90.10	8.98	19.81	4.612	727	2×10^{-9}	1.8×10^{55}
0.1	17.77	90.10	15.48	18.56	5.990	1739	2×10^{-9}	1.8×10^{56}
1	17.77	89.96	31.44	17.91	7.391	3974	2×10^{-9}	1.8×10^{57}
10	17.56	87.28	64.94	17.60	8.706	8405	2×10^{-9}	1.5×10^{58}
20	17.27	83.70	78.41	17.64	8.636	10^4	2×10^{-9}	5.2×10^{57}
30	16.99	80.25	86.05	17.83	7.970	10^4	2×10^{-9}	1.6×10^{57}
40	16.73	77.09	90.98	18.78	6.190	5523	2×10^{-8}	3.1×10^{56}

B violation which would erase the above mentioned baryon asymmetry. Furthermore, if we leave aside the electroweak symmetry breaking transition, which, within the standard model, does not seem to affect much the baryon asymmetry [40], and assuming normal adiabatic, radiation dominated evolution, the quark to photon ratio left by the Affleck-Dine baryogenesis should not be significantly changed down to the QCD transition considered here.

Therefore, we have taken several different ratios $\frac{n_q}{n_\gamma}(t_c)$, keeping in mind the necessary requirement $\frac{n_q}{n_\gamma}(t_f) \simeq 10^{-9}$ at the end of the transition. This requirement can be met, for moderate values of the baryonasymmetry, by changing slightly the value of λ. However, for larger values of $\frac{n_q}{n_\gamma}(t_c)$ the change in λ is more pronounced, in order to get enough expansion to dilute the initial baryon asymmetry to the desired value. The results with $B^{1/4} = 120$ MeV, $m_s = 100$ MeV and different values of λ are shown in table 1.

The first result which is striking in this table is the apparent proportionality between the initial quark number asymmetry $\frac{n_q}{n_\gamma}(t_c)$ and the number of quarks enclosed in a typical quark star. When $\frac{n_q}{n_\gamma}(t_c)$ grows from 0.01 to 10, $\mathcal{N}_q$ takes values corresponding to a quark content of 0.005 to 5 $M_\odot$. This seems natural: let us assume that the parameters B and m_s (remember that λ is no longer a free parameter) determine the geometrical repartition of bubbles, i.e. the mean distance between nucleation sites. Then, the more quarks you put initially in the volume which corresponds to a future quark object, the more quarks you will have in that object in the end. However, when $\frac{n_q}{n_\gamma}(t_c)$ is larger than 20, the number of quarks $\mathcal{N}_q$ does not increase anymore, but on the contrary it decreases. This means that it is impossible to form quark stars with gigantic masses, even with fine tuning, whereas in previous studies such fermion soliton stars could have a mass as high as 10^{12} $M_\odot$ [16, 41].

So far, we have assumed that the quark-hadron phase transition takes place between the quark phase and the QCD vacuum. In a subsequent study, we have considered the more realistic case with the transition between the quark plasma and a hadron phase containing the lightest hadrons, the pions. These contribute only to the phase $\xi = f_\pi$, by an additional term $-\frac{\pi^2}{30}T^4 + \frac{m_\pi^2}{8}T^2$ to the self-interaction potential $V_{T,\mu}(\xi = f_\pi)$,

which changes the expression of the overpressure (eq. 31) and the value of the critical temperature. Similarly, a term $x(\frac{\pi^2}{10}T^4 - \frac{m_\pi^2}{8}T^2)$ must be added to the energy density (eq. 32), thereby modifying Friedmann's equation and the energy conservation (eq. 33). The resulting calculations show that the qualitative features presented above remain unchanged, provided we slightly tune the value of λ. For example, with $B^{1/4} = 120$ MeV, $m_s = 100$ MeV and an initial baryon asymmetry $\frac{n_q}{n_\gamma}(t_c) = 10$, the baryon asymmetry at the end of the transition equals 10^{-9} if we take $\lambda = 17.76$, and the number of quarks enclosed in a typical quark star is $\mathcal{N}_q = 1.7 \times 10^{58}$, close to the value obtained when pions are neglected.

6. Conclusion

We have studied the quark-hadron phase transition within an effective model of QCD, the chiral quark model, including finite temperature and chemical potential. We would like to stress that this type of effective models is widely used in various fields of physics like the low energy hadron physics or relativistic heavy ion collisions. It is also worth noting that the so-called inflaton or dilaton scalar field models often used in cosmology are special cases of this model. In our work, all the parameters of the model are fixed by fits to the static properties of hadrons, and we have seen that in a reasonable range of the key parameters B, m_s and λ, quark plasma objects with very different masses can have been formed at the end of the transition, which are likely to survive until our epoch if their quark content is $\mathcal{N}_q \gtrsim 10^{40}$ (see the discussion and the references in [34]). The most interesting case regards the formation of a body with stellar mass, at a temperature much lower than the critical temperature of the transition. Such quark stars could be identified with some of the dark bodies detected by microlensing experiments in the halo of our galaxy [42]. A new feature of our calculations, independent of the size of the formed quark bodies, is the presence of a period of exponential expansion of the Universe during the transition which ends in a natural way with the transition. This growth, unexpected at such a temperature, could account for the observed baryon asymmetry in the Universe, if a high value of the quark to photon ratio had been left before the transition, at $T \gtrsim 100$ MeV by a baryogenesis mechanism *à la* Affleck-Dine, i.e. if we drop the usual assumption that fermion densities in the (very) early Universe were negligible: the entropy production by the released latent heat could have diluted the asymmetry down to the value it must have at $T \lesssim 10$ MeV to agree with the standard Big Bang nucleosynthesis. Actually, various models of entropy production have been designed for this purpose at very high energies by several authors [43], but our dilution mechanism is different from those earlier proposals in that it involves only the physics at the QCD scale.

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Figure captions

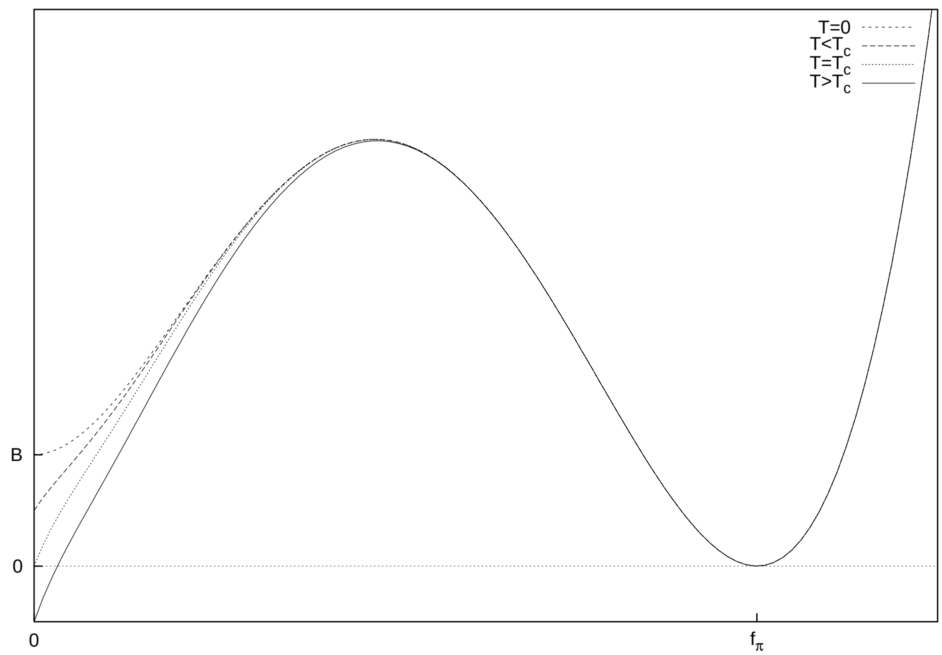

Figure 1. The effective potential $V_T(x)$. Solid curve corresponds to the potential at zero temperature; dashed curves to the potential at $T < T_c$, $T = T_c$ and $T > T_c$.

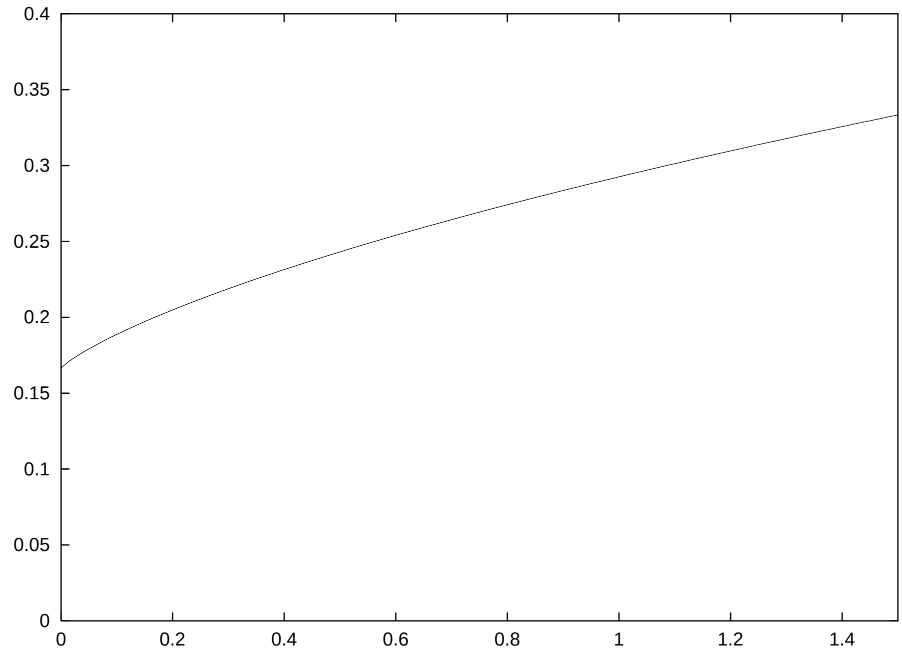

Figure 2. Graph of the integral $I(y)$ of equation (13).

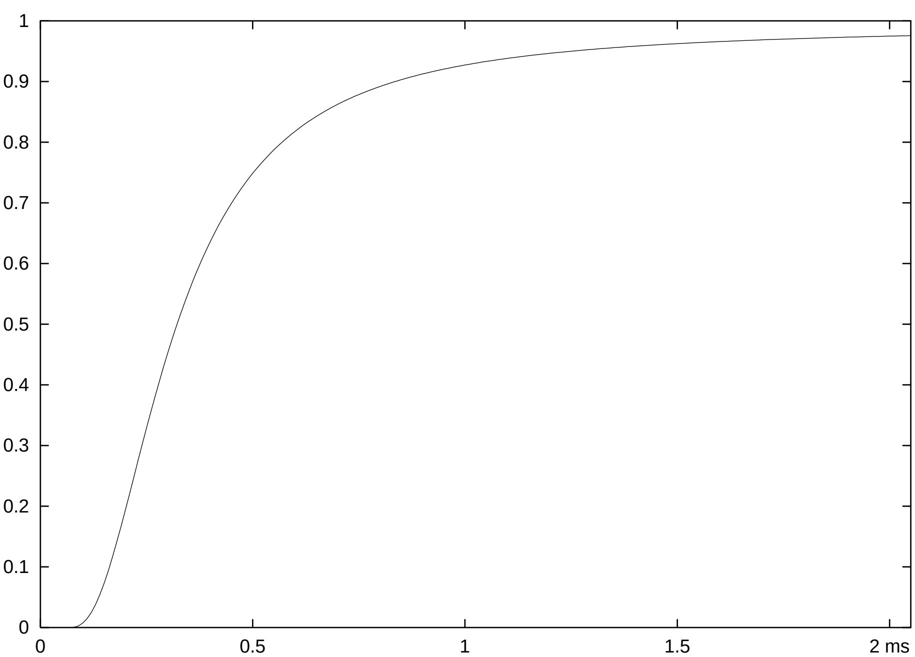

Figure 3. Evolution of the fraction of the Universe filled up with the phase $\xi = f_\pi$ during the transition in the case $B = (120\ \text{MeV})^4$, $m_s = 100$ MeV, $\lambda \simeq 17.77$.

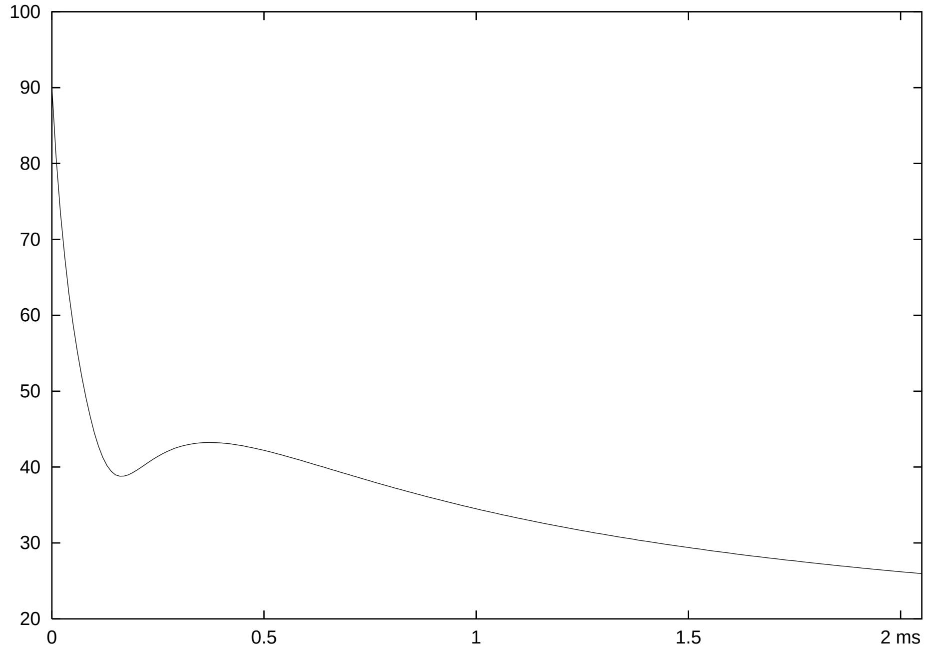

Figure 4. Evolution of temperature during the transition.

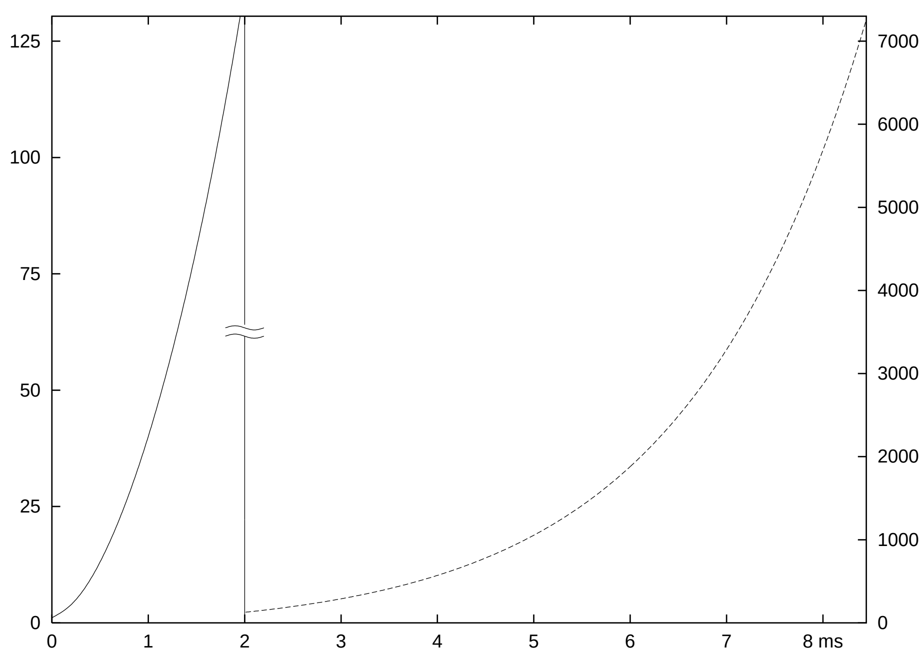

Figure 5. Evolution of the scale factor during the transition.